

Axioms¹

1. Problem # 1. (C level)

(a) $\mathbb{Q}_{2,3}$ = the set of all fractions, written in lowest terms, with denominator 2, or 3. Is $(\mathbb{Q}_{2,3}, +)$ closed? Explain.

(b) $\mathbb{Q}_{1,2,4}$ = the set of all fractions, written in lowest terms, with denominator 1, 2, or 4. Is $(\mathbb{Q}_{1,2,4}, +)$ closed? Explain.

2. Problem # 2. Let $a \oplus b = ab - b$ for all $a, b \in \mathbb{Z}$.

(a) Is (\mathbb{Z}, \oplus) closed? Explain.

(b) Is (\mathbb{Z}, \oplus) commutative? Explain.

(c) Is (\mathbb{Z}, \oplus) associative? Explain.

(d) Does (\mathbb{Z}, \oplus) have an identity element? Explain.

¹Abstract Algebra and Solution by Radicals, John E. Maxfield and Margaret W. Maxfield, Dover